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Tripletless Unification in the Conformal Window

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Abstract

A product SU(5) × Sp(4) grand unified model is proposed with no fundamental Higgs fields transforming under SU(5). Higgs doublets are instead embedded into a four dimensional representation of the Sp(4) gauge group, and hence there is no doublet-triplet splitting problem because there are no triplets. The Sp(4) group contains enough matter to lie in the conformal window, causing its gauge coupling to flow to a strongly-coupled infrared fixed-point at low energy, naturally preserving gauge coupling unification to percent level accuracy. Yukawa couplings, including the top, arise through dimension five operators that are enhanced by the large anomalous dimension of the Higgs fields. Proton decay mediated by dimension five operators is absent at the perturbative level. It reappears, however, non-perturbatively due to Sp(4) instantons but the rate is suppressed by a high power of the ratio of the dynamical scale to the unification scale. With gravity- or gaugino-mediated supersymmetry breaking, non-universal gaugino masses are predicted, satisfying specific one-loop renormalization group invariant relations. These predictions should be easily testable with the LHC and a linear collider.

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1 Introduction

The intersection of the three Standard Model (SM) gauge couplings at high energy with weak scale supersymmetry has long remained a tantalizing hint for a supersymmetric grand unified theory (GUT). Grand unification simplifies the matter content, by embedding matter into much simpler GUT representations, and simplifies the gauge group structure, by embedding the standard model into a simple GUT group. Minimal non-supersymmetric and supersymmetric simple group unification does, however, face a severe fine-tuning problem associated with the Higgs sector. Embedding Higgs doublets into GUT representations requires color triplet partners. The color triplet partners must be heavy, to prevent Higgs-mediated proton decay and preserve gauge coupling unification, while the the Higgs doublets must be light to obtain electroweak breaking at the weak scale. This is the infamous doublet-triplet splitting problem.

In supersymmetry, the doublet-triplet splitting problem is even more enigmatic because the fermionic partners to the color triplet Higgs fields lead to proton decay through dimension five operators at a rate somewhat faster than the present experimental bound. In minimal SU(5), for example, the mass scale of the color triplet Higgsinos must be several times larger than the GUT scale [1, 2]. There are several approaches to address this problem and a vast accompanying literature. (See for example in four dimensions [3, 4, 5, 6, 7, 8], extra dimensions [9, 10], deconstruction [11, 12], and string theory [13].)

Our proposal to solve doublet-triplet splitting is to dispose entirely of Higgs triplets by simply removing the complete GUT Higgs representation out of the theory. Of course Higgs doublet fields must somehow reappear in the model so that electroweak symmetry can be broken. The mechanism we employ to accomplish this is to embed the Standard Model into a diagonal subgroup of a GUT group and another simple group. Something similar was first footnoted by Yanagida [8, 14] who proposed $\mathrm{SU}(5) \times \mathrm{SU}(2)_H \times \mathrm{U}(1)_H$. (Later the proton decay signal of this model was examined in Ref. [15].) The idea is that the would-be Higgs fields begin their life as $\mathbf{2} + \mathbf{2}$ of $\mathrm{SU}(2)_H$, without the need of any color partners, and become ordinary Higgs doublets only after the product group diagonally breaks to the SM. This is what we call a "tripletless" GUT model.

This is similar in some respects to the missing partner mechanism where doublet-triplet splitting is accomplished by introducing additional partners whose sole purpose is to marry off the color triplet Higgs fields. Dimension five operators for proton decay are suppressed or forbidden by enforcing appropriate auxiliary symmetries, such as global $\mathrm{U}(1)_{PQ}$, that restrict the form of the allowed mass terms. Missing partner models are most easily constructed with product (GUT) gauge theories. The simplest model is based on $\mathrm{SU}(5) \times \mathrm{SU}(3)_H \times \mathrm{U}(1)_H$ proposed by Yanagida [8] and studied in detail in Refs. [16, 17, 14, 18, 19]. The partners of the color triplet Higgs fields are a $\mathbf{3} + \bar{\mathbf{3}}$ of $\mathrm{SU}(3)_H$ in the model. After the GUT breaks, the color triplet Higgs fields obtain mass by pairing off with these partners while the doublets do not acquire mass due to the lack of suitable partners. Missing partner models that marry off color triplets with other fields in the process of product group diagonal breaking we call "tripletfull" GUT models.

Tripletfull and tripletless product group models have the virtue that they completely solve the doublet-triplet splitting problem and can easily solve the triplet-induced dimension-5 proton decay problem. But they too face problems of their own creation. First, the gauge couplings do not generically unify. Instead, approximate unification emerges only if the hypergroups' couplings are large, greater than of order a few to obtain the intersection of the gauge couplings at the GUT scale to within a few percent. Large values of these couplings, however, is itself problematic. A strongly-coupled $\mathrm{U}(1)_H$ is a disaster, since the U(1) is not asymptotically free. The U(1) hits a Landau pole only slightly above the GUT scale, causing the effective theory to break down. Furthermore, the other hypercolor groups may also not be asymptotically free (the $\mathrm{SU}(2)_H$ coupling in the above tripletless model is an example). Finally, the $\mathrm{U}(1)_H$ factor also spoils the automatic prediction of charge quantization.

All the above problems can be avoided in models with the SU(5) $\times$ G gauge interaction with G being a simple, asymptotically-free group. Asymptotic freedom allows a large value of the gauge coupling at the GUT scale without any UV difficulties. This general approach, however, may have a coincidence problem. There is no dynamical explanation why the G group gauge coupling is large at the same scale as the GUT breaking scale (determined by other parameters in the superpotential), and this means a modest fine-tuning appears to be unavoidable.

In this paper we propose the first tripletless GUT model with all high energy groups asymptotically free based on SU(5) $\times$ Sp(4). We show that the Sp(4) gauge interaction has a strongly-coupled infrared fixed point that automatically solves the coincidence problem. The IR fixed point behavior also enhances the dimension-5 Yukawa couplings, allowing us to obtain the top Yukawa from a higher dimensional operator with an $\mathcal{O}$(few) coupling. Proton decay through dimension five operators is perturbatively forbidden, but reappears non-perturbatively at a suppressed level due to quantum effects of Sp(4). Given supersymmetry breaking communicated through gravity or gaugino mediation, non-universal gaugino masses are predicted, satisfying specific one-loop renormalization group invariant relations. These predictions should be easily testable with the LHC and a linear collider.

Finally, we remark that there is a tripletfull GUT model in this class of theories, the SO(10) $\times$ $\mathrm{SO}(6)_H$ model [20]. This theory, with 11 flavors transforming under $\mathrm{SO}(6)_H$, is within but right on the edge of the conformal window. If the SO(6) coupling is already at its IR fixed point at the Planck scale, there is no coincidence problem. However, this model does require numerous auxiliary fields to achieve SO(10) breaking. As we will see below, our tripletless model is much simpler in field content in part because higher dimensional operators receive a significant enhancement through the large anomalous dimensions of the fields.

2 The SU(5) $\times$ Sp(4) Model

The Higgs sector of our model consists of an Sp(4) gauge theory with six flavors, T_1 and T_2, Q^i and $\overline{Q}_i$ where $i = 1 \ldots 5$. SU(5) is a gauged subgroup of the global SU(12) flavor symmetry. The particle content of the model is summarized in Table 1. The tree level superpotential is given

	SU(5)	Sp(4)
Q^i_α	$\mathbf{5}$	$\mathbf{4}$
$\overline{Q}_{\alpha,i}$	$\overline{\mathbf{5}}$	$\mathbf{4}$
$T_{1,\alpha}$	$\mathbf{1}$	$\mathbf{4}$
$T_{2,\alpha}$	$\mathbf{1}$	$\mathbf{4}$

Table 1: The particle content of our SU(5) $\times$ Sp(4) tripletless product GUT model. Matter (not shown) is embedded into the usual $(\mathbf{10},\mathbf{1}) \oplus (\overline{\mathbf{5}},\mathbf{1})$ representations of SU(5).

by

$$\begin{aligned} W &= m(Q\cdot\overline{Q}) - \frac{1}{M_1}(Q\cdot\overline{Q})(Q\cdot\overline{Q}) - \frac{1}{M_2}(Q^i\cdot\overline{Q}_j)(Q^j\cdot\overline{Q}_i) - \frac{1}{M_3}(Q^i\cdot Q^j)(\overline{Q}_j\cdot\overline{Q}_i) + \cdots \\ &\quad + \frac{1}{M_T}(Q^i\cdot T_2)(\overline{Q}_i\cdot T_2)\ , \end{aligned} \tag{1}$$

where $i,j,\ldots$ represent SU(5) indices. We use the notation $(A^i\cdot B_j)$ to denote the Sp(4) contraction $A^i_\alpha B_{\beta,j}J^{\alpha\beta}$, where α,β represent the Sp(4) indices. The matrix $J^{\alpha\beta} = i\sigma_2\otimes\mathbf{1}_2$ is the invariant antisymmetric tensor of Sp(4). The contraction $(A\cdot B)$ without explicit SU(5) indices corresponds to $(A^i\cdot B_j)\delta^j_i$. We have assumed that T_1 does not appear in the superpotential, since it will become a pair of Higgs doublets after the GUT breaks. The extra field T_2 is necessary to avoid the Witten anomaly [21]. We omit all other possible operators as they will not affect our subsequent analysis.

Superpotential terms such as $(Q\cdot T_1)(\overline{Q}\cdot T_1)$ (and mixed terms involving T_1 and T_2) must not be written with order one coefficients because they induce a GUT scale μ-term for the Higgs fields. Symmetries can easily justify the lack of such terms. For example, impose a global U(1) symmetry under which T_1 has charge unity and neither of Q, $\overline{Q}$, nor T_2 are charged. Yukawa couplings are allowed with the charge assignment of $\mathbf{10} : -1/2$ and $\overline{\mathbf{5}} : -1/2$. This U(1) symmetry is, however, anomalous and broken by Sp(4) instantons. Nevertheless it is straightforward to modify the model by adding spectators charged under Sp(4) and U(1) so as to cancel the mixed anomaly. For example, adding N flavors of Sp(4) with a U(1) charge $-1/(2N)$ cancels the Sp(4)-Sp(4)-U(1) anomaly. The extra flavors will obtain mass if the U(1) symmetry is broken at high energy by vacuum expectation values (vevs) of fields ϕ and $\bar{\phi}$ with charge $1/N$ and $-1/N$. The lowest-dimensional gauge-invariant operator leading to a μ-term is $\phi^{2N}(Q\cdot T_1)(\overline{Q}\cdot T_1)/M^{2N-1}$, and can be suppressed to the weak scale or below by taking N large enough and/or the ratio $\langle\phi\rangle/M$ small enough.

We now analyze the moduli space of the theory. At the origin where $Q^i_\alpha = \overline{Q}_{\alpha,i} = 0$, the low energy effective theory is an Sp(4) gauge theory with one flavor that is known to generate a non-perturbative superpotential with a runaway direction and therefore no ground state. Hence, Q and $\overline{Q}$ must acquire vevs so that the GUT group breaks down [20]. Interestingly, as we see later, Q and $\overline{Q}$ also cannot acquire an expectation value in only one of its components as this would leave an Sp(2) theory with a dynamically generated run-away superpotential. Hence, Q and $\overline{Q}$ must acquire vevs for two components, uniquely selecting the vacuum with rank$\langle Q\rangle = 2$

that suggests the SM is the preserved diagonal subgroup gauge symmetry.[†]

First we analyze the theory classically, showing how doublet-triplet splitting is achieved. Later we discuss in detail the non-perturbative corrections. The potential is minimized with

$$Q = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ v & 0 & 0 & 0 \\ 0 & v & 0 & 0 \end{pmatrix} \ , \quad \overline{Q} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & v & 0 \\ 0 & 0 & 0 & v \end{pmatrix} \ , \quad T_1 = T_2 = 0 \ , \tag{2}$$

where

$$v = \sqrt{\frac{m M_1 M_2}{2M_1 + 4M_2}} \ . \tag{3}$$

The vanishing components of the matrices are ensured by the Sp(4) and SU(5) D-flat conditions. In the vacuum given by Eq. (2), SU(5) $\times$ Sp(4) is broken down to the standard model gauge group. The $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ electroweak group is the diagonal subgroup of SU(2) $\times$ U(1) in SU(5) and that in Sp(4). All of the pseudo-Goldstone components in Q, $\overline{Q}$, and T_2 acquire mass (as we show in detail below) while T_1 remains massless. T_1 becomes the Higgs doublet fields of the minimal supersymmetric standard model (MSSM). The doublet-triplet splitting problem is trivially solved as there are no colored Higgs particles.

Matching the SM gauge couplings $g_{1,2,3}$ to the high energy couplings g_5 [SU(5)] and g_4 [Sp(4)] we obtain the tree-level relations

$$\frac{1}{g_3^2} = \frac{1}{g_5^2} \quad , \quad \frac{1}{g_2^2} = \frac{1}{g_5^2} + \frac{2}{g_4^2} \quad , \quad \frac{1}{g_1^2} = \frac{1}{g_5^2} + \frac{6}{5g_4^2} \ . \tag{4}$$

The numerical factors are determined by the embedding of SU(2) $\times$ U(1) in Sp(4). Here g_1 is in the usual GUT normalization where $g_Y^2 = 3/5 g_1^2$. Obtaining gauge coupling unification at $\mathcal{O}(1\%)$ level requires the gauge coupling of Sp(4) to be $g_4 \gtrsim 7$ at the GUT scale.

Such a large coupling for Sp(4) implies that we must consider the non-perturbative effects of this strongly-coupled gauge interaction. The most convenient way to analyze the theory non-perturbatively is to first rewrite the superpotential Eq. (1) in terms of renormalizable interactions:

$$\begin{aligned} W &= m(Q^I \cdot \overline{Q}_I) + \lambda_{24} X_i^j (Q^i \cdot \overline{Q}_j) + \lambda_S S(Q \cdot \overline{Q}) + m_{24} X_j^i X_i^j + m_S S^2 + \tilde{m}^2 (S - \sqrt{15}\, \mathrm{tr}\, t^{24} X) \\ &\quad + k \tilde{H}^i (\overline{Q}_i \cdot T_2) + \bar{k} \overline{\tilde{H}}_i (Q^i \cdot T_2) + m_5 \tilde{H}^i \overline{\tilde{H}}_i \ . \end{aligned} \tag{5}$$

Here $I, J = 1, 2, 3$, and S, X, $\tilde{H}$, and $\overline{\tilde{H}}$ are massive SU(5) singlet, adjoint, fundamental and anti-fundamental fields, respectively, that do not acquire vevs. This superpotential is equivalent to Eq. (1) upon integrating out the heavy fields. Nevertheless, this superpotential shows that our model is simply a mass deformed Sp(4) gauge theory with six flavors. Integrating out the

[†]Note that the SM is uniquely chosen if the superpotential is minimal, i.e., Eq. (1) without the '$\cdots$' terms.

three massive flavors gives a low energy effective theory described by Q^A_α, $\overline{Q}_{A,\alpha}$, T_1 and T_2 with $A = 4, 5$ and spectators X, S, $\tilde{H}$ and $\overline{\tilde{H}}$ which can be thought of as mass deformations, i.e., the mass deformed three flavor theory. The non-perturbative effect of this theory is well-known to be a theory with quantum modified moduli space. By symmetries, holomorphy and various limits, we cannot write down a superpotential. The flavor symmetry suggests that the non-perturbative superpotential appears with a combination of $\mathrm{Pf}\, M$, where M is the meson fields made of two of Q, $\overline{Q}$, and T's, but it is forbidden by the non-anomalous R-symmetry as $R(M) = 0$ [22]. Since T_1 could only appear in the superpotential by the non-perturbative effect, mass terms for the Higgs-doublet fields are not generated even though there is no unbroken symmetry which protects the masslessness. In this low energy effective theory, the moduli space is modified by quantum effects. In particular, the classical constraint $\mathrm{Pf}\, M = 0$ is modified to $\mathrm{Pf}\, M = \Lambda^6$, where Λ is the dynamical scale of the three flavor Sp(4) theory. The constraint forces an SU(5)-singlet composite direction $M_{TT} \sim (T_1 \cdot T_2)$ to obtain a vev of the order of

$$\langle M_{TT} \rangle \sim \frac{\Lambda^6}{v^4} \ . \tag{6}$$

The other directions such as $(\bar{Q} \cdot T_1)(Q \cdot T_2)$ are protected against acquiring vevs by the superpotential terms with $\tilde{H}$ and $\overline{\tilde{H}}$. As we will see, this non-perturbative effect in Eq. (6) is important in our discussion of proton decay. Finally, notice that the vacuum with rank $\langle Q \rangle = 2$ is uniquely chosen dynamically. A similar analysis with above shows that, in the vacuum with rank$\langle Q \rangle = 1(0)$, the effective theory is an Sp(4) gauge theory with two (one) flavors. Such small N_f theories are known to dynamically generate a run-away superpotential for the massless flavors causing the vacuum to be unstable.

The most important non-perturbative quantum effect of our strongly-coupled Sp(4) is the existence of an infrared fixed point. Above the scale m, the theory is an Sp($2N_c$) gauge theory with $2N_c = 4$ and $N_f = 6$, which is known to be in the conformal window $\frac{3}{2}(N_c + 1) < N_f < 3(N_c+1)$ with a non-trivial strongly-coupled fixed-point of the renormalization group [23]. There are several important consequences. First, the gauge coupling is naturally large, $\mathcal{O}(4\pi)$, which ensures the gauge couplings unify to excellent accuracy. Next, there is no coincidence between the scale where the gauge coupling becomes strong and the GUT breaking scale. In particular, they are both determined the supersymmetric mass, m, that effectively sets the dynamical scale $\Lambda \sim m$. Finally, the coupling constants in the superpotential obtain a large enhancement from the large anomalous dimensions of the Sp(4) fields. The anomalous dimension of the meson operator is

$$\gamma^* = 1 - 3\frac{N_c + 1}{N_f} = -\frac{1}{2} \tag{7}$$

which enhances the coefficients of the operators at low energy such as

$$m(\mu) = m(\mu_0) \left(\frac{\mu}{\mu_0}\right)^{-\frac{1}{2}} \ , \quad \frac{1}{M_X(\mu)} = \frac{1}{M_X(\mu_0)} \left(\frac{\mu}{\mu_0}\right)^{-1} \ . \tag{8}$$

where M_X stands for M_1, M_2, M_3, and M_T in Eq. (1), and μ_0 is the scale where the gauge coupling constant reaches $\mathcal{O}(4\pi)$. Clearly, the non-renormalizable operators in Eq. (1) are not

necessarily suppressed by the Planck scale but could be only suppressed by the GUT scale, if the Sp(4) group is already strongly coupled at the Planck scale.

The Yukawa interactions between matter and Higgs fields arise from higher dimensional operators

$$W_{\text{Yukawa}} = \frac{1}{M_U}\epsilon_{ijklm}\mathbf{10}^{ij}\mathbf{10}^{kl}(Q^m \cdot T_1) + \frac{1}{M_D}\mathbf{10}^{ij}\bar{\mathbf{5}}_i(\overline{Q}_j \cdot T_1) \ , \tag{9}$$

where $\mathbf{10}$ and $\bar{\mathbf{5}}$ are the matter fields and the generation indices have been suppressed. For the top quark, the Yukawa coupling evaluated at the GUT scale is roughly $\lambda_t \sim 0.5-0.7$ (depending on $\tan\beta$) which appears to require an unnaturally small M_U. However, the large anomalous dimensions of the Sp(4) fields causes an enhancement of up to $\mathcal{O}[(M_{\text{Pl}}/M_{\text{GUT}})^{1/2}] \sim 10$. We need only a slightly large coefficient of the Planck suppressed operator, of order 5, to obtain the observed top Yukawa coupling at the weak scale. It is also possible to enhance the top Yukawa coupling by introducing a pair of massive fields with $\mathbf{5}+\bar{\mathbf{5}}$ of SU(5) with renormalizable interactions that, after integrating out these fields, generates a top Yukawa. We will not pursue this (admittedly ad hoc) extension of our model, but note nevertheless that it does not invalidate our analysis of the masslessness of the Higgs doublet fields.

The enhancement of the mass parameter suggests that the original mass scale of the model is lower than the GUT scale. If the neutrino masses are explained by the seesaw mechanism, the natural scale for the right-handed neutrino masses are the lower original scale because there is no enhancement. It provides a natural explanation for the smallness of the right-handed neutrino masses which is necessary to reproduce the observed neutrino masses.

Since there are no colored Higgs fields, dimension five operators leading to proton decay are absent at the classical level. However, non-perturbative effects can reintroduce these operators with power-suppressed coefficients. After integrating out the heavy fields, we obtain operators such as

$$\frac{\lambda_u\lambda_d}{M_T m^2 v^2}\epsilon_{ijklm}\mathbf{10}^{ij}\mathbf{10}^{kl}\mathbf{10}^{mn}\bar{\mathbf{5}}_n(T_1 \cdot T_2)(T_1 \cdot T_2) \ , \tag{10}$$

where λ_u and λ_d are the Yukawa coupling constants. Since the combination of $(T_1 \cdot T_2)$ acquires a vev from the quantum modified constraint, Eq. (6), we obtain

$$W_{\text{eff}} = \frac{f_u f_d \Lambda^{12}}{M_T m^2 v^{10}} \ \epsilon_{ijklm}\mathbf{10}^{ij}\mathbf{10}^{kl}\mathbf{10}^{mn}\bar{\mathbf{5}}_n \ . \tag{11}$$

The constraint from the proton lifetime requires the effective colored Higgs mass of $(M_T m^2 v^{10})/\Lambda^{12}$ to be larger than about 10^{17} GeV (see, e.g., Ref. [2]). This is easily satisfied by taking the dynamical scale Λ to be slightly smaller than the GUT scale v. This is naturally realized in our model since the GUT scale is $\mathcal{O}(\sqrt{mM})$ while $\Lambda \sim m$.

3 Threshold Corrections to Gauge Coupling Unification

Here we discuss the threshold corrections to gauge coupling unification. The matter content of our model, decomposed into fields transforming under the SM group,

$$Q = \left[\begin{array}{cc} (\mathbf{3},\mathbf{2})_{-5/6} & (\mathbf{3},\mathbf{2})_{1/6} \\ (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{1},\mathbf{1})_0 & (\mathbf{1},\mathbf{3})_1 \oplus (\mathbf{1},\mathbf{1})_1 \end{array} \right] \tag{12}$$

$$\overline{Q} = \left[\begin{array}{cc} (\overline{\mathbf{3}},\mathbf{2})_{-1/6} & (\overline{\mathbf{3}},\mathbf{2})_{5/6} \\ (\mathbf{1},\mathbf{3})_{-1} \oplus (\mathbf{1},\mathbf{1})_{-1} & (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{1},\mathbf{1})_0 \end{array} \right] \tag{13}$$

$$T_1 = \left[\, (\mathbf{1},\mathbf{2})_{-1/2} \;\; (\mathbf{1},\mathbf{2})_{1/2} \, \right] \tag{14}$$

$$T_2 = \left[\, (\mathbf{1},\mathbf{2})_{-1/2} \;\; (\mathbf{1},\mathbf{2})_{1/2} \, \right] \tag{15}$$

where our notation for the fields is to write their quantum numbers as $[\mathrm{SU}(3)_c, \, \mathrm{SU}(2)_L]_{\mathrm{U}(1)_Y}$. Upon $\mathrm{SU}(5) \times \mathrm{Sp}(4)$ breaking to $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, there are $24+10-12 = 22$ broken generators. These massive gauge bosons include the X, Y's of SU(5) that eat the $(\mathbf{3},\mathbf{2})_{-5/6}$ and $(\overline{\mathbf{3}},\mathbf{2})_{5/6}$ fields; the broken Sp(4) generators that eat the $(\mathbf{1},\mathbf{3})_{\pm 1}$; and the broken linear combinations of the remaining Sp(4) generators and the $\mathrm{SU}(2) \times \mathrm{U}(1)$ subgroup of SU(5) that absorb one $(\mathbf{1},\mathbf{3})_0$ and one $(\mathbf{1},\mathbf{1})_0$ fields. T_1 becomes the a pair of SM Higgs doublets that survive to the weak scale. This leaves several pseudo-Goldstone fields that acquire mass $\mathcal{O}(m)$ only from the higher dimensional operators in Eq. (1). Specifically, the uneaten $(\mathbf{1},\mathbf{3})_0$ field acquires a mass of order $v^2/M_1 + v^2/M_2$; the $(\mathbf{3},\mathbf{2})_{1/6}$ and $(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$ fields pair up with a mass of order $v^2/M_2 + v^2/M_3$; finally the $(\mathbf{1},\mathbf{1})_{\pm 1}$ fields pair up with a mass of order v^2/M_3. Finally, the second set of Higgs doublets from T_2 pair up with a mass of order v^2/M_T.

Above the scale of the pseudo-Goldstone fields, but below the GUT breaking scale, the one-loop beta function coefficients of the SM couplings are shifted by an amount

$$\Delta b_1 = 2 \quad , \quad \Delta b_2 = 6 \quad , \quad \Delta b_3 = 2 \, . \tag{16}$$

In a weakly-coupled product gauge theory, taking the pseudo-Goldstone fields to have roughly the same mass scale $v^2/M \sim 10^{-2} v$, these shifts in the beta functions induce a very significant threshold correction to gauge coupling unification.‡ This would-be disaster is averted due to the large anomalous dimensions of the strongly-coupled Sp(4) that provide a large enhancement to the mass parameters of the model, as we found in Eq. (8). Let us consider the maximum enhancement, when the Sp(4) gauge interaction is conformal up to the Planck scale. Starting with dimension-5 operators that are $1/M_{\mathrm{Pl}}$ suppressed, the conformal enhancement causes $1/M_{\mathrm{Pl}}(M_{\mathrm{GUT}}/M_{\mathrm{Pl}})^{-1} \to 1/M_{\mathrm{GUT}}$, and therefore the masses of the pseudo-Goldstone fields are increased up to roughly the GUT scale. Obviously no large threshold correction from these fields is expected. This required assuming the Sp(4) group was conformal up to the Planck scale, which is consistent with maximizing the enhancement of the top Yukawa. We sketch the spectrum of this theory in a purely weakly-coupled case and in the strongly-coupled, conformal up to the Planck scale case in Fig. 1.

‡The higher dimensional operators could have different coefficients and therefore the pseudo-Goldstone fields could have different masses. In particular, the $(\mathbf{1},\mathbf{1})_{\pm 1}$ fields could be easily made much lighter than the other pseudo-Goldstone fields, mitigating some of effects of this threshold correction.

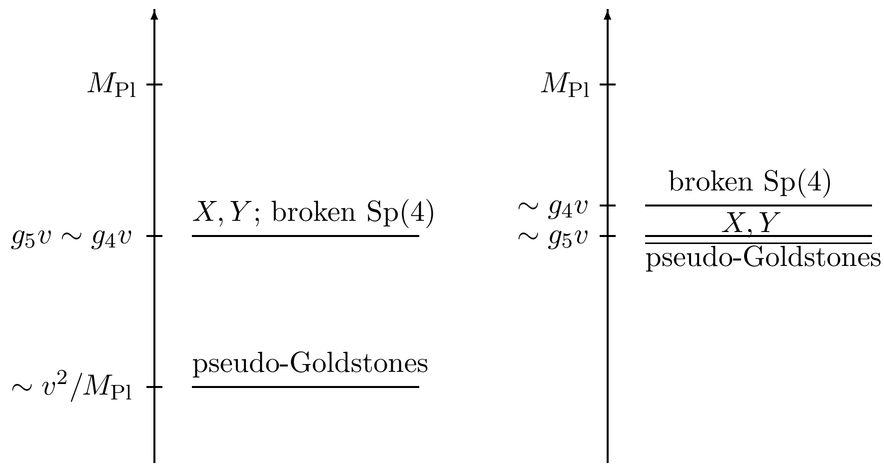

Figure 1: A sketch of the mass spectrum is shown for a weakly-coupled SU(5) $\times$ Sp(4) theory (left side), and for our model with a strongly-coupled Sp(4) that is conformal up to the Planck scale with the maximal anomalous dimension enhancement (right side).

With Sp(4) in the conformal window, the correction to SM gauge coupling from tree-level matching, Eqs. (4), is also negligible. This assumes of course that the infrared fixed-point value $g_4 \gtrsim 7$. This cannot be verified perturbatively, but as an aside we remark that the two-loop correction to the Sp(4) gauge coupling running is the same as the one-loop correction when $g_4 = 2\pi$, which roughly satisfies our constraint. A large Sp(4) coupling also causes the massive gauge bosons to be split in mass, namely $m_{X,Y} \sim g_5 v \equiv M_{\rm GUT}$ while the the broken Sp(4) generators have a mass of order $g_4 v$. The effect of this threshold correction is equivalent to a (fake) redefinition of the coupling g_4 in the tree-level matching, and so causes no new issues.

4 Experimental Signals

The dominance of proton decay through dimension six operators is the smoking gun of tripletless models, as well as most proposals to solve the doublet-triplet splitting problem in supersymmetry. In our model, the GUT scale (scale of X, Y gauge bosons) is at or slightly higher than the scale in minimal SU(5), which means the proton decay rate through X, Y exchange is expected to be even (slightly) longer than would be predicted in minimal SU(5) GUT. Proton decay through dimension-five operators is possible but extremely unlikely due to the severe sensitivity to the ratio Λ/v. This does not bode well for testing our model through proton decay experiments.

Supersymmetric product group models do have one very interesting signal: gaugino mass non-universality [24]. Suppose that the mechanism of supersymmetry breaking is UV sensitive, leading to gaugino masses for both SU(5) gauginos and Sp(4) gauginos through operators such as

$$\int d^2\theta \, \left[f_5(S) W^5_\alpha W^{5,\alpha} + f_4(S) W^4_\alpha W^{4,\alpha} \right] \ , \tag{17}$$

where $f_5(S)$ and $f_4(S)$ are gauge kinetic functions that acquire expectation value for both the scalar and auxiliary components. These operators appear in supergravity mediation as well as gaugino mediation [25]. The scalar components of the gauge kinetic functions give the gauge coupling constants $[f_5]_S = 1/(2g_5^2)$ and $[f_4]_S = 1/(2g_4^2)$, whereas the gaugino masses are given by $[f_5]_F = M_5/g_5^2$ and $[f_4]_F = M_4/g_4^2$. This leads to tree-level matching relations for the SM gauginos

$$\frac{M_3}{\alpha_3} = \frac{M_5}{\alpha_5} \tag{18}$$

$$\frac{M_2}{\alpha_2} = \frac{M_5}{\alpha_5} + 2\frac{M_4}{\alpha_4} \tag{19}$$

$$\frac{M_1}{\alpha_1} = \frac{M_5}{\alpha_5} + \frac{6}{5}\frac{M_4}{\alpha_4} \; . \tag{20}$$

The deviations from gaugino mass universality are proportional to M_4/α_4. If the auxiliary components of the gauge kinetic functions f_5 and f_4 are comparable, the deviation from universality is expected to be large, although this is UV-dependent.

Notice that the three SM gaugino masses depend on only two high energy parameters. This implies the prediction

$$\frac{M_1}{\alpha_1} - \frac{3}{5}\frac{M_2}{\alpha_2} - \frac{2}{5}\frac{M_3}{\alpha_3} = 0 \; , \tag{21}$$

that is renormalization group invariant to one-loop, and therefore valid at the weak scale to within a few percent accuracy [26]. This appears to be a unique prediction of our SU(5) × Sp(4) product GUT model. Certainly there is no such prediction for the product GUT models involving SU(5) × $\mathrm{SU}(3)_H$ × $\mathrm{U}(1)_H$ since the U(1) normalization is unknown. However, our relation happens to be precisely the same as predicted by the SO(10) × SO(6) tripletfull model [27][§]. This happens despite, for example, $\mathrm{SU}(2)_L$ coming from a diagonal subgroup of SU(5) × Sp(4) in our model, whereas it comes from purely SO(10) in the model of Ref. [20]. This means that to unambiguously test our model against other product group models we must examine the individual ratios of gaugino masses. With our model, we find the ratios

$$1 \;<\; \left.\frac{M_2/\alpha_2}{M_1/\alpha_1}\right|_{\mathrm{SU(5)\times Sp(4)}} \;<\; \frac{5}{3} \tag{22}$$

$$1 \;>\; \left.\frac{M_3/\alpha_3}{M_2/\alpha_2}\right|_{\mathrm{SU(5)\times Sp(4)}} \;>\; 0 \tag{23}$$

where the left(right)-hand side is obtained in the limit $M_4 \to 0$ ($M_5 \to 0$). In models in which SU(2) does *not* arise as a diagonal subgroup, e.g. tripletfull models such as the SO(10) × SO(6) model, these ratios always lies in the range

$$1 \;>\; \left.\frac{M_2/\alpha_2}{M_1/\alpha_1}\right|_{\mathrm{tripletfull}} \;>\; 0 \tag{24}$$

$$1 \;<\; \left.\frac{M_3/\alpha_3}{M_2/\alpha_2}\right|_{\mathrm{tripletfull}} \;<\; \infty \tag{25}$$

[§]We thank Y. Nomura for pointing this out to us.

where again the left(right)-hand side is obtained in the limit the hypercolor gaugino mass(es) go to zero (the GUT gaugino mass goes to zero). The Bino-to-Wino mass ratio may be the best bet experimentally, since it is expected to be tested to high accuracy at a linear collider by looking at the endpoint distribution in chargino decay [28]. Mass ratios involving the gluino inevitably involve larger QCD uncertainties, but may still yield important information.

5 AdS interpretation?

Our model was constructed purely as a four-dimensional strongly-coupled supersymmetric product group theory. Nevertheless, it is intriguing to speculate about a possible dual five-dimensional AdS interpretation to our CFT theory. We emphasize that there is by no means a certainty that such an AdS interpretation exists, but it is interesting to sketch the outcome of reversing the AdS/CFT dictionary. Our analysis is in the spirit of [29], and perhaps most closely linked with the CFT interpretation [30] (see also [31]) of warped Higgsless theories [32].

The region between Planck and the GUT scales translates into a slice of AdS space in which the endpoints are interpreted as Planck and GUT branes. Our CFT has a large global symmetry, part of which is gauged. The weakly gauged SU(5) subgroup of the SU(12) global symmetry translates into the bulk of AdS containing the SU(5) gauge symmetry. Matter fields in our CFT are elementary fields, corresponding to Planck brane-localized fields. Since the Higgs doublets arise as composites of the CFT, the AdS interpretation is that they are fields localized to the GUT brane. SU(5) is broken by the strong dynamics of the CFT, corresponding to boundary conditions on the GUT brane that break SU(5) down to the SM gauge group. The AdS interpretation of doublet-triplet splitting therefore appears qualitatively similar to the flat space model of Ref. [10]. Namely, Higgs doublets can be present on the GUT brane because the GUT brane respects just the SM group.

On the CFT side, we find several perturbatively stable vacua, such as $Q, \overline{Q}$'s containing no vevs, that translate into SU(5) preserving boundary conditions of the GUT brane. However, using the improved knowledge of strongly-coupled supersymmetric theories, we showed that this SU(5) invariant vacuum was non-perturbatively unstable (developing a run-away superpotential). This suggests the perturbatively stable SU(5) preserving even boundary conditions on the AdS side may actually be unstable non-perturbatively, developing a run-away superpotential on the GUT brane.

The remainder of the SU(12)/SU(5) global symmetry of our CFT is explicitly broken by Planck scale effects, giving mass for the pseudo-Goldstones through $1/M_{\rm Pl}$-suppressed operators. These operators are enhanced by the large anomalous dimension of the CFT to $1/M_{\rm GUT}$. An equivalent 4D description corresponds to rewriting the higher dimensional operators in terms of renormalizable interactions with spectator fields that get integrated out, as we showed in Eq. (5). The AdS interpretation is that these auxiliary fields are bulk fields with various wavefunction profiles that lead to both higher dimensional operators involving CFT fields as well as matter-CFT couplings, i.e., Yukawa couplings.

Finally, our CFT is a small N_c theory whose breaking effects, the massive gauge bosons associated with broken Sp(4) generators, correspond in the AdS picture to strongly-coupled Kaluza-Klein resonances of the compactified spacetime. This dual 5D interpretation is, therefore, not obviously useful as a calculational tool for us. It could however provide insight into translating strongly-coupled AdS theories into small N 4D conformal field theories that, for supersymmetric cases at least, we have better knowledge.

6 Discussion

We have constructed a tripletless product SU(5) $\times$ Sp(4) grand unified model by throwing out both the Higgs doublets and triplets from SU(5). This resolved the fine-tuning and rapid proton decay problems associated with Higgs triplets. Higgs doublets are present in the model, arising not from SU(5) but instead a four dimensional representation of the Sp(4) gauge group. The Sp(4) group has enough flavor to be in the conformal window, and therefore its gauge coupling naturally flows to a strongly-coupled infrared fixed-point at low energy. This ensures gauge coupling unification to percent level accuracy, but without the coincidence problem of why the Sp(4) group got strong at the GUT scale (it was, in fact, always strong). Yukawa couplings, including the top, arise through dimension five operators that are enhanced by the large anomalous dimension of the Higgs fields. Proton decay mediated by dimension five operators, while absent at the perturbative level, arises non-perturbatively from the Sp(4) gauge interaction but is sufficiently suppressed. With gravity or gaugino mediation, non-universal gaugino masses are predicted, satisfying specific one-loop renormalization group invariant relations. These predictions can be tested at the LHC and a linear collider.

It is amusing to note that some recent approaches to mediating supersymmetry breaking with flavor-blind scalar masses also employ supersymmetric gauge theories in the conformal window. Luty and Sundrum proposed a hidden sector with a CFT whose large anomalous dimensions caused a suppression of the direct couplings of the hidden sector to the visible sector [33], thereby allowing anomaly mediation to dominate. Separately, Nelson and Strassler proposed using a CFT to generate Yukawa hierarchies as well as suppressing soft-breaking parameters [34]. It is tempting to consider models in which the CFT is used for both product GUT breaking and supersymmetry breaking, as this may yield firmer predictions for the soft supersymmetry breaking parameters. However, various hurdles must be surmounted. In Luty-Sundrum, for example, a much larger hierarchy between the strong and confinement scales is needed to get enough conformal sequestering, and it certainly appears problematic to obtain Higgs fields from the CFT (as we do) that would be part of the hidden sector (for Luty-Sundrum).

Finally, it is interesting to speculate about other product GUT theories in the conformal window. It is straightforward to extend our model to an SO(10) $\times$ Sp(4), but additional auxiliary fields transforming under SO(10) are needed to break this group down the SM. Another possibility is to consider a tripletfull model based on SU(5) $\times$ Sp(6). This model contains fundamental Higgs doublets and triplets, where the triplets marry off with a $\mathbf{6} = \mathbf{3} + \bar{\mathbf{3}}$ representation of Sp(6). More matter is needed so that the Sp(6) theory resides in the conformal window,

but otherwise the analysis proceeds analogously to the analysis of the SU(5) $\times$ Sp(4) model presented above. Yukawa couplings are simply the ordinary MSSM marginal operators. One important difference, however, is that the Higgs doublets can acquire a supersymmetric mass term from non-perturbative effects (instantons) of Sp(6). This is again suppressed by a ratio of the dynamical scale to the vev, but now this ratio has to be numerically order 10 to ensure this contribution is at or below the weak scale. This may be somewhat tricky to achieve simultaneous with a sufficiently strongly-coupled Sp(6) theory that is needed to avoid large tree-level threshold corrections as well as providing a sufficient enhancement of the masses of the pseudo-Goldstone fields. In any case, there is clearly a larger class of product GUT theories that live in the conformal window, solving the doublet-triplet splitting problem as well as naturally preserving gauge coupling unification.

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